Lecture-2

Electromagnetic wave propagation: Wave propagation in lossy dielectrics, plane waves in lossless dielectrics, plane wave in free space, plain waves in good conductors, power and the pointing vector, reflection of a plain wave in a normal incidence.

Electromagnetic waves

For the electric field E,

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

$$= -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\mu_o \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

or,

$$\nabla^2 \vec{E} - \mu_o \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

i.e. wave equation with $v^2 = 1/\mu_o\varepsilon$

Electromagnetic waves

Similarly for the magnetic field $$\nabla^2 \vec{B} - \mu_o \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

i.e. wave equation with $v^2 = 1/\mu_o\varepsilon$

In free space, **$\varepsilon = \chi\, \varepsilon_o = \varepsilon_o$** **($\chi = 1$)**

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

$c = 3.0 \times 10^8$ m/s

Electromagnetic waves

In a dielectric medium, $\chi = n^2$ **and** $\varepsilon = \chi\, \varepsilon_o = n^2\, \varepsilon_o$

$$v = \frac{1}{\sqrt{\mu_o \varepsilon}} = \frac{1}{n\sqrt{\mu_o \varepsilon_o}} = \frac{c}{n}$$

Electromagnetic waves: Phase relations

The solutions to the wave equations,

$$\nabla^2 \vec{E} - \mu_o \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \nabla^2 \vec{B} - \mu_o \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

can be plane waves,

$$\vec{E} = \vec{E}_o e^{i\left(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t\right)}$$

$$\vec{B} = \vec{B}_o e^{i\left(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t + \delta\right)}$$

Electromagnetic waves: Phase relations

- Using plane wave solutions one finds that,

$$\nabla \times \vec{E} = i\left(\vec{k} \times \vec{E}\right)$$

$$and,$$

$$\frac{\partial \vec{B}}{\partial t} = -i\omega\vec{B}$$

- Consequently

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$or,$$

$$\vec{k} \times \vec{E} = \omega\vec{B}$$

i.e. B is perpendicular to the Plane formed by k and E !!

Electromagnetic waves: Phase relations

- Since also

$$\nabla \cdot \vec{E} = i\vec{k} \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = i\vec{k} \cdot \vec{B} = 0$$

- k ⊥ E and k ⊥ B (transverse wave)
- Thus, k, E and B are mutually perpendicular vectors
- Moreover,

$$\vec{k} \times \vec{E} = \left|\vec{k}\right|\left|\vec{E}\right|\hat{B} = \omega B\hat{B}$$

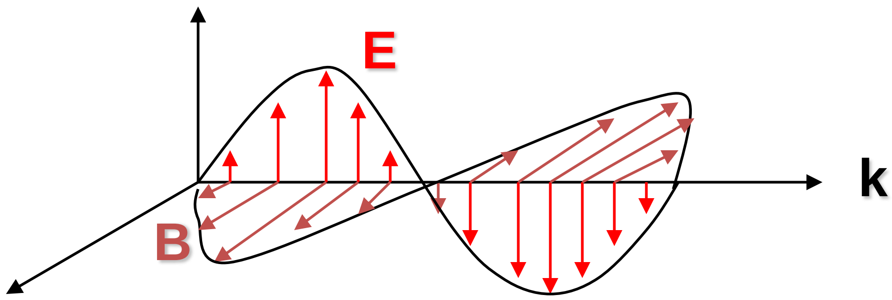
Electromagnetic waves: Phase relations

Thus E and B are in phase since,

$$E_o e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} = cB_o e^{i\left(\vec{k}\cdot\vec{r}-\omega t+\delta\right)}$$

requires that

$$e^{i\delta} = 1 \quad \delta = 0, 2\pi, \ldots$$



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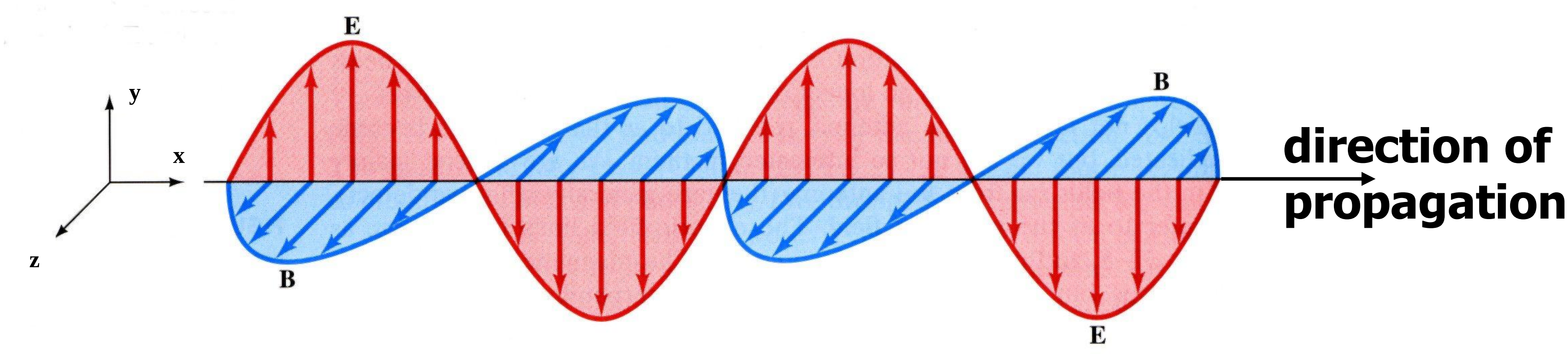
Electromagnetic waves are transverse waves, but are not mechanical waves (they need no medium to vibrate in).

Therefore, electromagnetic waves can propagate in free space.

At any point, the magnitudes of $\vec{E}$ and $\vec{B}$ (of the wave shown) depend only upon x and t, and not on y or z. A collection of such waves is called a plane wave.



Manipulation of Maxwell's equations leads to the following plane wave equations for $\vec{E}$ and $\vec{B}$:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z(x,t)}{\partial t^2}$$

These equations have solutions:

$$E_y = E_{max} \sin(kx - \omega t)$$

$$B_z = B_{max} \sin(kx - \omega t)$$

E_{max} and B_{max} are the electric and magnetic field amplitudes

where $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$, and $f\lambda = \frac{\omega}{k} = c.$

You can verify this by direct substitution.

E_{max} and B_{max} in these notes are sometimes written by others as E_0 and B_0.

You can also show that

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$E_{max}\ k \cos(kx - \omega t) = B_{max}\ \omega \cos(kx - \omega t)$$

$$\boxed{\frac{E_{max}}{B_{max}} = \frac{E}{B} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.}$$

At every instant, the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light.

Summary of Important Properties of Electromagnetic

The solutions of Maxwell's equations are wave-like with both $\vec{E}$ and $\vec{B}$ satisfying a wave equation.

$$E_y = E_{max}\sin(kx - \omega t)$$

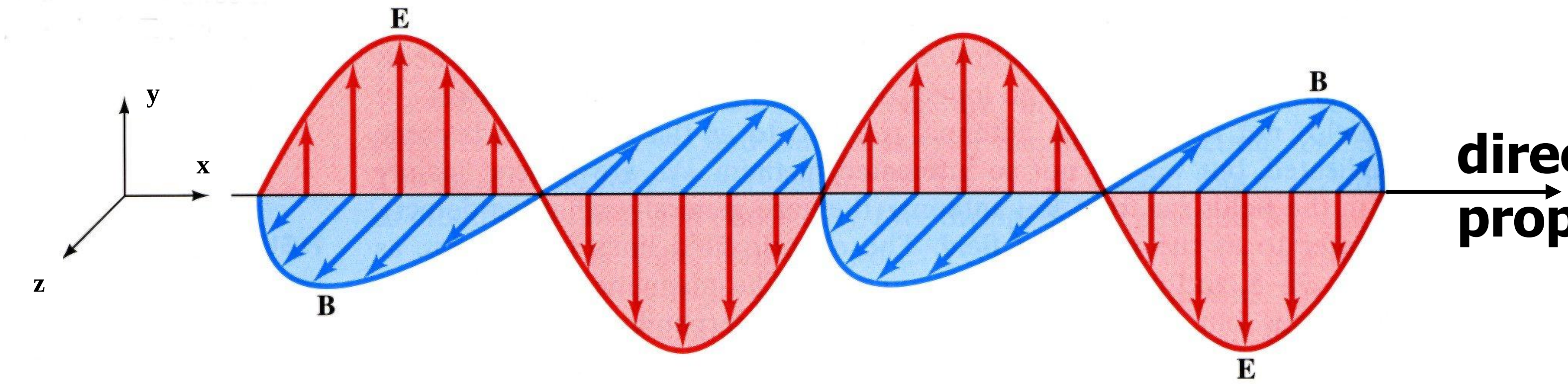
$$B_z = B_{max}\sin(kx - \omega t)$$

Electromagnetic waves travel through empty space with the speed of light $c = 1/(\mu_0\varepsilon_0)^{1/2}$.

E_{max} and B_{max} are the electric and magnetic field amplitudes.

Summary of Important Properties of Electromagnetic

The components of the electric and magnetic fields of plane EM waves are perpendicular to each other and perpendicular to the direction of wave propagation. The latter property says that EM waves are transverse



The magnitudes of $\vec{E}$ and $\vec{B}$ in empty space are related by

E/B = c.

$$\frac{E_{max}}{B_{max}} = \frac{E}{B} = \frac{\omega}{k} = c$$

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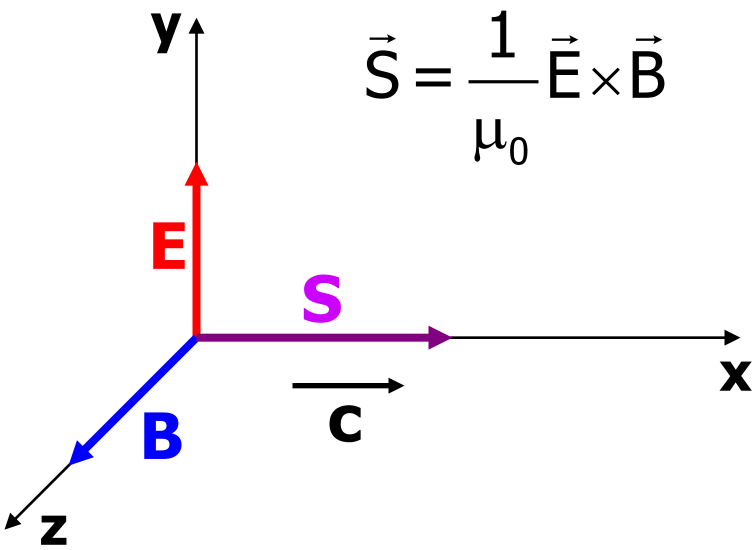
Energy Carried by Electromagnetic Waves

Electromagnetic waves carry energy, and as they propagate through space they can transfer energy to objects in their path. The rate of flow of energy in an electromagnetic wave is described by a vector $\vec{S}$, called the Poynting vector.*

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

The magnitude S represents the rate at which energy flows through a unit surface area perpendicular to the direction of wave propagation.

Thus, S represents power per unit area. The direction of $\vec{S}$ is along the direction of wave propagation. The units of $\vec{S}$ are $J/(s\cdot m^2) = W/m^2$.



$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

For an EM wave $\left|\vec{E}\times\vec{B}\right| = EB$

so $S = \frac{EB}{\mu_0}.$

Because B = E/c we can write

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}.$$

These equations for S apply at any instant of time and represent the instantaneous rate at which energy is passing through a unit area.

$$S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

EM waves are sinusoidal.

$$E_y = E_{max} \sin(kx - \omega t)$$

$$B_z = B_{max} \sin(kx - \omega t)$$

The average of S over one or more cycles is called the wave intensity I.

The time average of $\sin^2(kx - \omega t)$ is ½, so

$$I = S_{average} = \langle S \rangle = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{cB_{max}^2}{2\mu_0}$$

The magnitude of $\vec{S}$ is the rate at which energy is transported by a wave across a unit area at any instant:

$$S = \left(\frac{\text{energy}/\text{time}}{\text{area}} \right)_{\text{instantaneous}} = \left(\frac{\text{power}}{\text{area}} \right)_{\text{instantaneous}}$$

Thus,

$$I = \langle S \rangle = \left(\frac{\text{energy}/\text{time}}{\text{area}} \right)_{\text{average}} = \left(\frac{\text{power}}{\text{area}} \right)_{\text{average}}$$

Note: $S_{average}$ and <S> mean the same thing!

Energy Density

The energy densities (energy per unit volume) associated with electric field and magnetic fields are:

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad\qquad u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Using B = E/c and $c = 1/(\mu_0\varepsilon_0)^{1/2}$ we can write

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{\left(E/c\right)^2}{\mu_0} = \frac{1}{2}\frac{\mu_0\varepsilon_0 E^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E^2$$

$$\boxed{u_B = u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}}$$

$$u_B = u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}$$

For an electromagnetic wave, the instantaneous energy density associated with the magnetic field equals the instantaneous energy density associated with the electric field.

Hence, in a given volume the energy is equally shared by the two fields. The total energy density is equal to the sum of the energy densities associated with the electric and magnetic fields:

$$u = u_B + u_E = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$u = u_B + u_E = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

When we average this instantaneous energy density over one or more cycles of an electromagnetic wave, we again get a factor of ½ from the time average of $\sin^2(kx - \omega t)$.

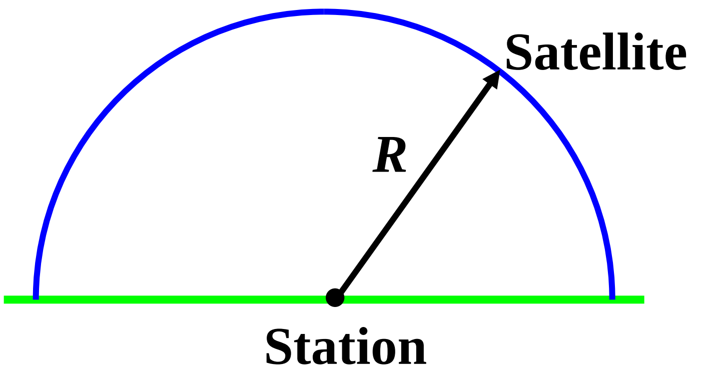
$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_{max}^2 \ , \ \langle u_B \rangle = \frac{1}{4}\frac{B_{max}^2}{\mu_0},$$ **and** $$\boxed{\langle u \rangle = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{B_{max}^2}{\mu_0}}$$

Recall $S_{average} = \langle S \rangle = \frac{1}{2}\frac{E_{max}^2}{\mu_0 c} = \frac{1}{2}\frac{cB_{max}^2}{\mu_0}$ **so we see that** $\langle S \rangle = c\langle u \rangle$.

The intensity of an electromagnetic wave equals the average energy density multiplied by the speed of light.

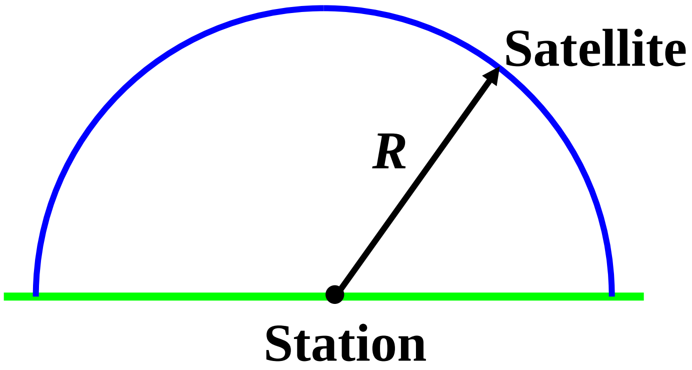
Example: a radio station on the surface of the earth radiates a sinusoidal wave with an average total power of 50 kW. Assuming the wave is radiated equally in all directions above the ground, find the amplitude of the electric and magnetic fields detected by a satellite 100

All the radiated power passes through the hemispherical surface* so the average power per unit area (the intensity) is



$$I = \left(\frac{\text{power}}{\text{area}}\right)_{\text{average}} = \frac{P}{2\pi R^2} = \frac{\left(5.00\times10^{4}\ \text{W}\right)}{2\pi\left(1.00\times10^{5}\ \text{m}\right)^2} = 7.96\times10^{-7}\ \text{W/m}^2$$

$$I = \langle S \rangle = \frac{1}{2}\frac{E_{max}^2}{\mu_0 c}$$



$$E_{max} = \sqrt{2\mu_0 c I}$$

$$= \sqrt{2\left(4\pi \times 10^{-7}\right)\left(3 \times 10^{8}\right)\left(7.96 \times 10^{-7}\right)}$$

$$= 2.45 \times 10^{-2}\ \text{V}/\text{m}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{\left(2.45 \times 10^{-2}\ \text{V}/\text{m}\right)}{\left(3 \times 10^{8}\ \text{m/s}\right)} = 8.17 \times 10^{-11}\ \text{T}$$

Example: for the radio station in the example on the previous two slides, calculate the average energy densities associated with the electric and magnetic

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_{max}^2 \qquad \langle u_B \rangle = \frac{1}{4}\frac{B_{max}^2}{\mu_0}$$

$$\langle u_E \rangle = \frac{1}{4}\left(8.85\times10^{-12}\right)\left(2.45\times10^{-2}\right)^2 \qquad \langle u_B \rangle = \frac{1}{4}\frac{\left(8.17\times10^{-11}\right)^2}{\left(4\pi\times10^{-7}\right)}$$

$$\langle u_E \rangle = 1.33\times10^{-15}\,\frac{J}{m^3} \qquad \langle u_B \rangle = 1.33\times10^{-15}\,\frac{J}{m^3}$$

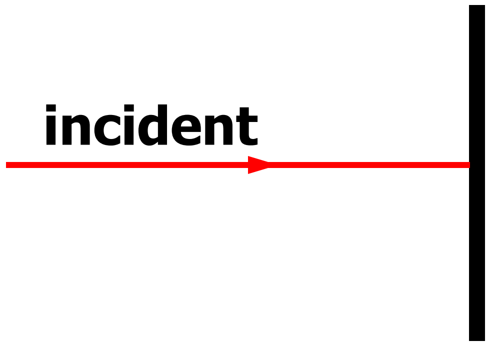
Momentum and Radiation Pressure

EM waves carry linear momentum as well as energy. When this momentum is absorbed at a surface pressure is exerted on that surface.

If we assume that EM radiation is incident on an object for a time Δt and that the radiation is entirely absorbed by the object, then the object gains energy ΔU in time Δt.

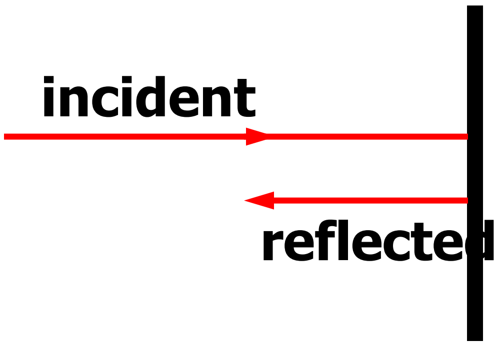
Maxwell showed that the momentum change of the object is then:

$$|\Delta p| = \frac{\Delta U}{c} \quad \text{(total absorption)}$$



The direction of the momentum change of the object is in the direction of the incident radiation.

If instead of being totally absorbed the radiation is totally reflected by the object, and the reflection is along the incident path, then the magnitude of the momentum change of the object is twice that for total absorption.



$$|\Delta p| = \frac{2\Delta U}{c} \quad \text{(total reflection along incident path)}$$

The direction of the momentum change of the object is again in the direction of the incident radiation.

Radiation Pressure

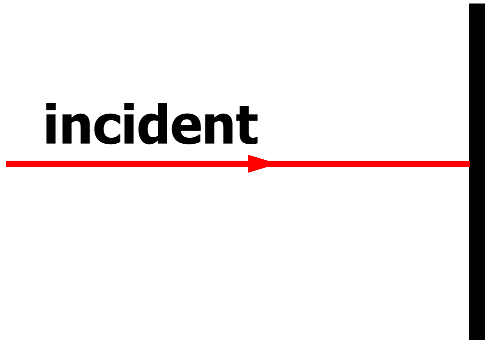
The radiation pressure on the object is defined as the force per unit area:

$$P = \frac{F}{A}$$

From Newton's 2nd Law (F = dp/dt) we have:

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$

For total absorption, $\Delta p = \dfrac{\Delta U}{c}$



So $P = \dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{1}{A}\dfrac{d}{dt}\left(\dfrac{U}{c}\right) = \dfrac{1}{c}\left(\dfrac{dU/dt}{A}\right) = \dfrac{S}{c}$

(Equations on this slide involve magnitudes of vector quantities.)

This is the instantaneous radiation pressure in the case of total absorption:

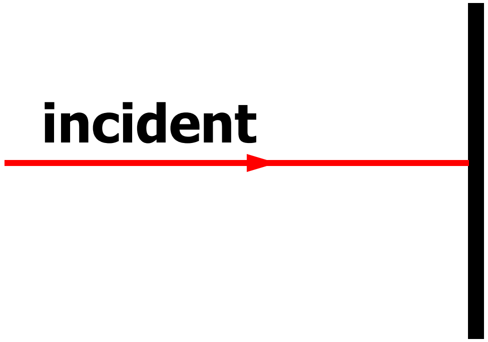
$$P = \frac{S}{c}$$

For the average radiation pressure, replace S by $\langle S \rangle = S_{avg} = I$:

$$P_{rad} = \frac{S_{average}}{c} = \frac{I}{c}$$

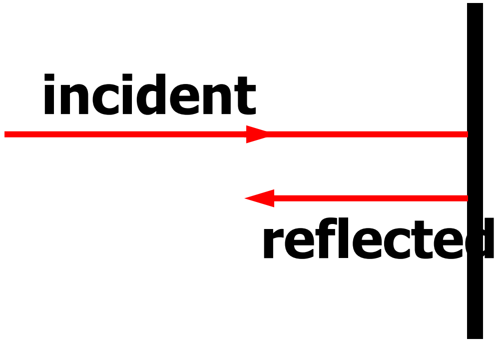
Electromagnetic waves also carry momentum through space with a momentum density of $S_{average}/c^2 = I/c^2$. This is not on your equation sheet but you have special permission to use it in tomorrow's homework, if necessary.

$$P_{rad} = \frac{I}{c}$$ (total absorption)



Following similar arguments it can be shown that:

$$P_{rad} = \frac{2I}{c}$$ (total reflection)



Example: a satellite orbiting the earth has solar energy collection panels with a total area of 4.0 m^2. If the sun’s radiation is incident perpendicular to the panels and is completely absorbed find the average solar power absorbed and the average force associated with the radiation pressure. The intensity (I or $S_{average}$) of sunlight prior to passing through the earth’s

$$\text{Power} = IA = \left(1.4\times10^{3}\ \text{W}/\text{m}^2\right)\left(4.0\ \text{m}^2\right) = 5.6\times10^{3}\,\text{W} = 5.6\ \text{kW}$$

Assuming total absorption of the radiation:

$$P_{rad} = \frac{S_{average}}{c} = \frac{I}{c} = \frac{\left(1.4\times10^{3}\ \text{W}/\text{m}^2\right)}{\left(3\times10^{8}\ \text{m}/\text{s}\right)} = 4.7\times10^{-6}\,\text{Pa}$$

$$F = P_{rad}A = \left(4.7\times10^{-6}\ \text{N}/\text{m}^2\right)\left(4.0\ \text{m}^2\right) = 1.9\times10^{-5}\,\text{N}$$

New starting equations from this lecture:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$S_{average} = \frac{1}{2}\frac{E_{max}^2}{\mu_0 c} = \frac{1}{2}\frac{cB_{max}^2}{\mu_0}$$

$$\frac{E_{max}}{B_{max}} = \frac{E}{B} = c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$u_B = u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}$$

$$k = \frac{2\pi}{\lambda},\ \ \omega = 2\pi f,\ \ f\lambda = \frac{\omega}{k} = c$$

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{B_{max}^2}{\mu_0}$$

$$|\Delta p| = \frac{\Delta U}{c}\ \text{ or }\ \frac{2\Delta U}{c}$$

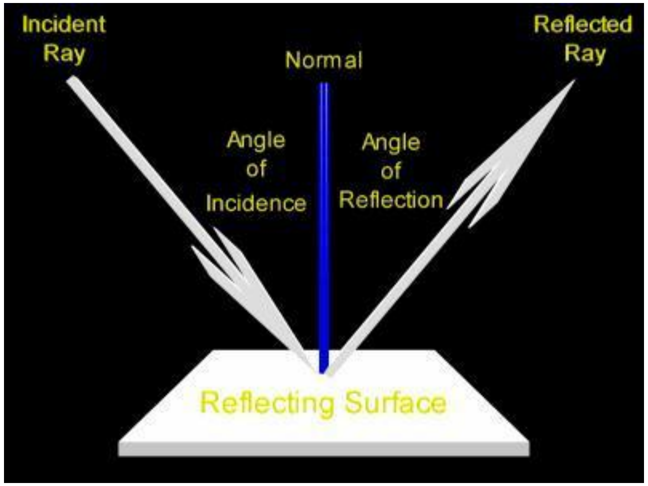
$$P_{rad} = \frac{I}{c}\ \text{ or }\ \frac{2I}{c}$$

There are even more on your starting equation sheet; they are derived from the above!

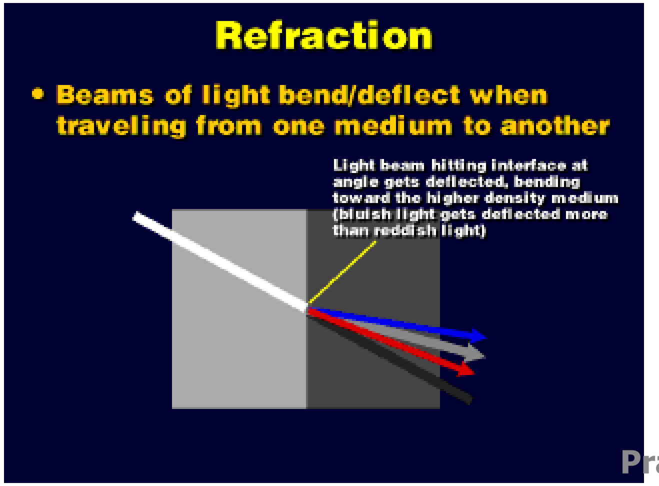
Electromagnetic waves:

Interact with matter in four ways:

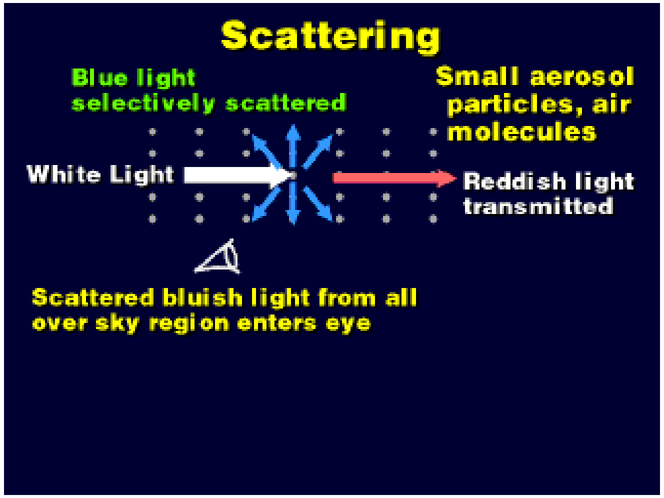
Reflection:



Refraction:



Scattering:



Diffraction: